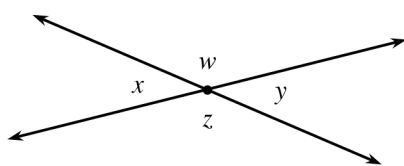


ANGLE PAIR RELATIONSHIPS

8.3.2

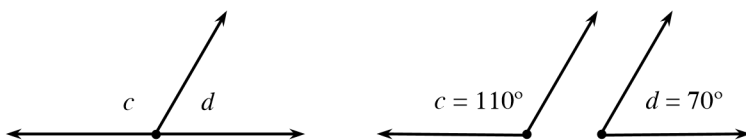
Properties of Angle Pairs

Intersecting lines form four angles. The pairs of angles across from each other are called vertical angles. The measures of vertical angles are equal.



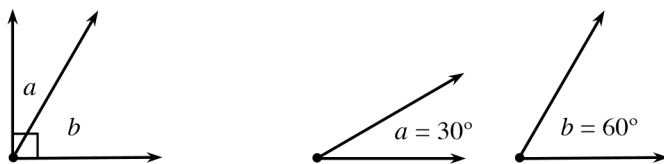
$\angle x$ and $\angle y$ are vertical angles
 $\angle w$ and $\angle z$ are vertical angles

If the sum of the measures of two angles is exactly 180° , then the angles are called supplementary angles.



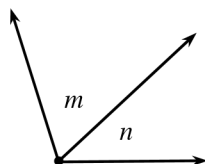
$\angle c$ and $\angle d$ are supplementary angles

If the sum of the measures of two angles is exactly 90° , then the angles are called complementary angles.



$\angle a$ and $\angle b$ are complementary angles

Angles that share a vertex and one side but have no common interior points (that is, do not overlap each other) are called adjacent angles.

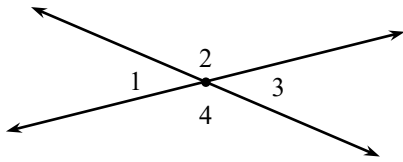


$\angle m$ and $\angle n$ are adjacent angles

For additional information, see the Math Notes box in Lesson 8.3.2 of the *Core Connections, Course 2* text.

Example 1

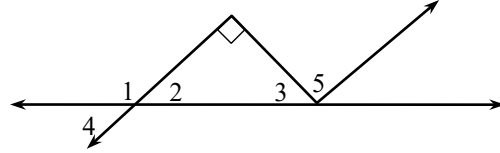
Find the measure of the missing angles if $m\angle 3 = 50^\circ$.



- $m\angle 1 = m\angle 3$ (vertical angles)
 $\Rightarrow m\angle 1 = 50^\circ$
- $\angle 2$ and $\angle 3$ (supplementary angles)
 $\Rightarrow m\angle 2 = 180^\circ - 50^\circ = 130^\circ$
- $m\angle 2 = m\angle 4$ (vertical angles)
 $\Rightarrow m\angle 4 = 130^\circ$

Example 2

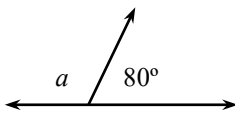
Classify each pair of angles below as vertical, supplementary, complementary, or adjacent.



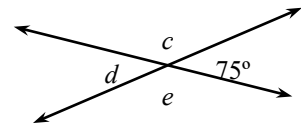
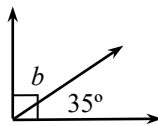
- a. $\angle 1$ and $\angle 2$ are adjacent and supplementary
- b. $\angle 2$ and $\angle 3$ are complementary
- c. $\angle 3$ and $\angle 5$ are adjacent
- d. $\angle 1$ and $\angle 4$ are adjacent and supplementary
- e. $\angle 2$ and $\angle 4$ are vertical

Problems

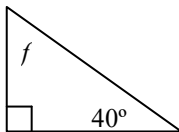
Find the measure of each angle labeled with a variable.



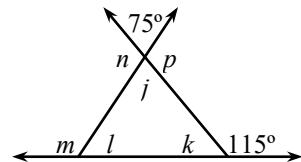
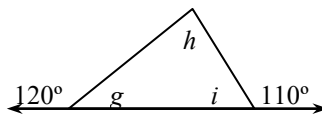
1. 2. 3.



4. 5.



6.



Answers

1. $m\angle a = 100^\circ$

2. $m\angle b = 55^\circ$

3. $m\angle c = 105^\circ$

$m\angle d = 75^\circ$

$m\angle e = 105^\circ$

4.

$m\angle f = 50^\circ$ 5. $m\angle g = 60^\circ$ 6. $m\angle j = 75^\circ$ $m\angle k =$
 65° $m\angle h = 50^\circ$ $m\angle l = 40^\circ$ $m\angle m = 140^\circ$ $m\angle i =$
 70° $m\angle n = 105^\circ$ $m\angle p = 105^\circ$

Math 1 and Math 2
Day 3 Alternative Assignment
Please do work on separate paper

ELIMINATION METHOD

6.3.1 – 6.3.3

Another method for solving systems of equations is the **Elimination Method**. This method is particularly convenient when both equations are in standard form (that is, $ax + by = c$). To solve this type of system, we can rewrite the equations by focusing on the coefficients. (The coefficient is the number in front of the variable.)

See problem 6-80 in the textbook for an additional explanation of the Elimination Method.

For additional information, see the Math Notes boxes in Lesson 6.3.3. For additional examples and more problems solving systems using multiple methods, see the Checkpoint 9 Materials.

Example 1

Solve: $x - y = 2$

$$2x + y = 1$$

Recall that you are permitted to add the same expression to both sides of an equation. Since $x - y$ is equivalent to 2 (from the first equation), you are permitted to add $x - y$ to one side of the second equation, and 2 to the other side, then solve.

$$\begin{array}{r} 2x + y = 1 \\ +x - y \quad + 2 \\ \hline 3x \quad = 3 \\ x = 1 \end{array}$$

Note that this was an effective way to eliminate y and solve for x because $-y$ and y eliminated each other.

$$\begin{array}{l} 2x + y = 1 \\ \text{Since } x = 1, \\ 2(1) + y = 1 \\ 2 + y = 1 \\ y = -1 \end{array}$$

Now substitute the value of x in either of the original equations to solve for y .

The solution is $(1, -1)$, since $x = 1$ and $y = -1$ make both of the original equations true. Always check your solution by substituting the x - and y -values back into *both* of the original equations to verify that you get true statements.

Example 2

Solve: $3x+6y=24$

$$3x+y=-1$$

Notice that both equations contain a $3x$ term. We can rewrite $3x + y = -1$ by multiplying both sides by -1 , resulting in $-3x + (-y) = 1$. Now the two equations have terms that are opposites: $3x$ and $-3x$. This will be useful in the next step because $-3x + 3x = 0$.

Since $-3x + (-y)$ is equivalent to 1 , we can add $-3x + (-y)$ to one side of the equation and add 1 to the other side.

$$\begin{array}{r} 3x + 6y = 24 \\ -3x + (-y) + 1 \\ \hline 5y = 25 \\ y = 5 \end{array}$$

Notice how the two opposite terms, $3x$ and $-3x$, eliminated each other, allowing us to solve for y .

Then substitute the value of y into either of the original equations to solve for x .

$$\begin{array}{r} 3x + 6(5) = 24 \\ 3x + 30 = 24 \\ 3x = -6 \\ x = -2 \end{array}$$

The solution is $(-2, 5)$. Always check your solution by substituting the solution back into *both* of the original equations to verify that you get true statements.

Example 3

Solve: $x+3y=7$

$$4x-7y=-10$$

To use the Elimination Method, one of the terms in one of the equations needs to be opposite of the corresponding term in the other equation. For example, in the system above, there are no terms that are opposite. However, if the first equation is multiplied by -4 , then the equations will have $4x$ and $-4x$. The first equation now looks like this: $-4(x + 3y = 7) \rightarrow -4x + (-12y) = -28$. When multiplying, be sure to multiply all the terms on *both* sides of the equation. With the first equation rewritten, the system of equations now looks like this:

$$-4x + (-12y) = -28$$

$$4x - 7y = -10$$

Since $4x - 7$ is equivalent to -10 , they can be added either side of the first equation:

$$\begin{array}{r} -4x + (-12y) = -28 \\ + \quad 4x - 7y \quad -10 \\ \hline \quad -19y \quad = -38 \\ \quad \quad y \quad = 2 \end{array}$$

Now any of the equations can be used to solve for x : The solution to the system of equations is $(1, 2)$.

$$\begin{array}{l} \text{Since } 4x - 7y = -10 \text{ and } y = 2, \\ 4x - 7(2) = -10 \\ 4x - 14 = -10 \\ 4x = 4 \\ x = 1 \end{array}$$

Example 4

If multiplying one equation by a number will not make it possible to eliminate a variable, multiply both equations by different numbers to get coefficients that are the same or opposites.

$$\begin{array}{l} \text{Solve: } 8x - 7y = 5 \\ \quad \quad 3x - 5y = 9 \end{array}$$

One possibility is to multiply the first equation by 3 and the second equation by -8 . The resulting terms $24x$ and $-24x$ will be opposites.

$$\begin{array}{l} 24x - 21y = 15 \\ -24x + 40y = -72 \end{array}$$

$$\begin{array}{l} 3(8x - 7y = 5) \Rightarrow 24x - 21y = 15 \\ -8(3x - 5y = 9) \Rightarrow -24x + 40y = -72 \end{array}$$

The system of equations is now:

This system can be solved by adding equivalent expressions (from the second equation) to the first equation:

$$\begin{array}{r} 24x - 21y = 15 \\ + \quad -24x + 40y \quad -72 \\ \hline \quad 19y = -57 \\ \quad \quad y = -3 \end{array}$$

Then, solving for x , the solution is $(-2, -3)$.

Example 5

Not all systems of equations have a solution. If solving the system results in an equation that is not true, then there is **no solution**. The graph of a system of two linear equations that has no solutions is two parallel lines; there is no point of intersection. See the Math Notes box in Lesson 6.4.1 for additional information.

$$\text{Solve: } y = 7 - 3x$$

$$3x + y = 10$$

Replace y in the second equation with $7 - 3x$.

The resulting equation is never true.
There is no solution to this system of equations.

$$3x + y = 10$$

$$3x + (7 - 3x) = 10$$

$$3x - 3x + 7 = 10$$

$$7 \neq 10$$

Example 6

There may also be **infinitely many solutions**. If solving the system results in an equation that is always true, then there are infinitely many solutions (all of the points on both lines when graphed). This graph would appear as a single line for the two equations.

Solve: $y = 4 - 2x$

$$-4x - 2y = -8$$

Substitute $4 - 2x$ in the second equation for y .

$$-4x - 2y = -8$$

$$-4x - 2(4 - 2x) = -8$$

$$-4x - 8 + 4x = -8$$

$$-8 = -8$$

This statement is always true. There are infinitely many solutions to this system of equations.

SUMMARY OF METHODS TO SOLVE SYSTEMS

Method	This Method is Most Efficient When	Example
Equal Values	Both equations in $y =$ form.	$y = x - 2$ $y = -2x + 1$
Substitution	One variable is alone on one side of one equation.	$y = -3x - 1$ $3x + 6y = 24$
Elimination: Add to eliminate one variable.	Equations in standard form with opposite coefficients.	$x + 2y = 21$ $3x - 2y = 7$
Elimination: Multiply one equation to eliminate one variable.	Equations in standard form. One equation can be multiplied to create opposite terms.	$x + 2y = 3$ $3x + 2y = 7$
Elimination: Multiply both equations to eliminate one variable.	When nothing else works. In this case you could multiply the first equation by 3 and the second equation by -2 , then add to eliminate the opposite terms.	$2x - 5y = 3$ $3x + 2y = 7$

Problems

- | | | | | | |
|-----|-------------------------|-----|--------------------------------|-----|----------------------------|
| 1. | $2x+y=6$
$-2x+y=2$ | 2. | $-4x+5y=0$
$-6x+5y=-10$ | 3. | $2x-3y=-9$
$x+y=-2$ |
| 4. | $y-x=4$
$2y+x=8$ | 5. | $2x-y=4$
$\frac{1}{2}x+y=1$ | 6. | $-4x+6y=-20$
$2x-3y=10$ |
| 7. | $6x-2y=-16$
$4x+y=1$ | 8. | $6x-y=4$
$6x+3y=-16$ | 9. | $2x-2y=5$
$2x-3y=3$ |
| 10. | $y-2x=6$
$y-2x=-4$ | 11. | $4x-4y=14$
$2x-4y=8$ | 12. | $3x+2y=12$
$5x-3y=-37$ |

Answers

- | | | | | | |
|-----|-------------|-----|----------------------|-----|---------------------------|
| 1. | (1, 4) | 2. | (5, 4) | 3. | (-3, 1) |
| 4. | (0, 4) | 5. | (2, 0) | 6. | infinitely many solutions |
| 7. | (-1, 5) | 8. | $(-\frac{1}{6}, -5)$ | 9. | (4.5, 2) |
| 10. | no solution | 11. | $(3, -\frac{1}{2})$ | 12. | (-2, 9) |

MULTIPLYING BINOMIALS

1.2.3

Two ways to determine the area of a rectangle are: as the product of its (height) and (base) or as the sum of the areas of individual pieces of the rectangle. For a given rectangle these two areas must be the same, so **area as a product = area as a sum**. Algebra tiles, and later, area models help students multiply expressions in a visual, concrete manner.

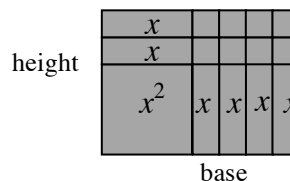
For additional information, see the Math Notes box in Lesson 1.3.1.

Example 1: Using Algebra Tiles

The algebra tile pieces $x^2 + 6x + 8$ are arranged into a rectangle as shown at right. The area of the rectangle can be written as the **product** of its base and height or as the **sum** of its parts.

$$\underbrace{(x+4)}_{\text{base}} \underbrace{(x+2)}_{\text{height}} = \underbrace{x^2 + 6x + 8}_{\text{area}}$$

area as a **product** area as a **sum**



Example 2: Using Area Models

An area model allows the problem to be organized in the same way as the first example, but without drawing the individual tiles. The rectangle does not have to be drawn accurately or to scale.

Multiply $\underbrace{(2x+1)}_{\text{base}} \underbrace{(x-3)}_{\text{height}}$.

$$\begin{array}{|c|c|} \hline -3 & \\ \hline x & \\ \hline 2x & +1 \\ \hline \end{array} \Rightarrow \begin{array}{|c|c|} \hline -3 & -6x \\ \hline x & 2x^2 \\ \hline 2x & +1 \\ \hline \end{array} \Rightarrow (2x+1)(x-3) = 2x^2 - 5x - 3$$

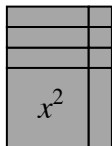
area as a product area as a sum

Note that the factors can be written in either order, so $(x-3)(2x+1) = 2x^2 - 5x - 3$ is alternate way to write the equation.

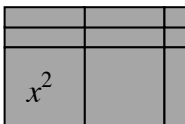
Problems

Write an equation showing **area as a product** equals **area as a sum**.

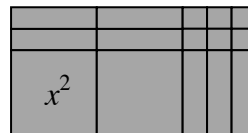
1.



2.



3.



4.

$$\begin{array}{r|cc} x & x^2 & 3x \\ -5 & -5x & -15 \\ \hline & x & +3 \end{array}$$

5.

$$6 \begin{array}{r|cc} & 18y & -12x \\ \hline & 3y & -2x \end{array}$$

6.

$$\begin{array}{r|cc} 3y & 3xy & 12y \\ -2 & -2x & -8 \\ \hline & x & +4 \end{array}$$

Multiply.

7. $(3x + 2)(2x + 7)$

8. $(2x - 1)(3x + 1)$

9. $(2x)(x - 1)$

10. $(2y - 1)(4y + 7)$

11. $(y - 4)(y + 4)$

12. $(y)(x - 1)$

13. $(3x - 1)(x + 2)$

14. $(2y - 5)(y + 4)$

15. $(3y)(x - y)$

16. $(3x - 5)(3x + 5)$

17. $(4x + 1)^2$

18. $(x + y)(x + 2)$

19. $(2y - 3)^2$

20. $(x - 1)(x + y + 1)$

21. $(x + 2)(x + y - 2)$

Answers

1. $(x + 1)(x + 3) = x^2 + 4x + 3$

2. $(x + 2)(2x + 1) = 2x^2 + 5x + 2$

3. $(x + 2)(2x + 3) = 2x^2 + 7x + 6$

4. $(x - 5)(x + 3) = x^2 - 2x - 15$

5. $6(3y - 2x) = 18y - 12x$

6. $(x + 4)(3y - 2) = 3xy - 2x + 12y - 8$

7. $6x^2 + 25x + 14$

8. $6x^2 - x - 1$

9. $2x^2 - 2x$

10. $8y^2 + 10y - 7$

11. $y^2 - 16$

12. $xy - y$

13. $3x^2 + 5x - 2$

14. $2y^2 + 3y - 20$

15. $3xy - 3y^2$

16. $9x^2 - 25$

17. $16x^2 + 8x + 1$

18. $x^2 + 2x + xy + 2y$

19. $4y^2 - 12y + 9$

20. $x^2 + xy - y - 1$

21. $x^2 + xy + 2y - 4$

LAWS OF EXPONENTS

3.1.1 and 3.1.2

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic **laws of exponents** are listed here.

- (1) $x^a \cdot x^b = x^{a+b}$ Examples: $x^3 \cdot x^4 = x^7$; $2^7 \cdot 2^4 = 2^{11}$
- (2) $\frac{x^a}{x^b} = x^{a-b}$ Examples: $\frac{x^{10}}{x^4} = x^6$; $\frac{2^4}{2^7} = 2^{-3}$
- (3) $(x^a)^b = x^{ab}$ Examples: $(x^4)^3 = x^{12}$; $(2x^3)^5 = 2^5 \cdot x^{15} = 32x^{15}$
- (4) $x^0 = 1$ Examples: $2^0 = 1$; $(-3)^0 = 1$; $(\frac{1}{4})^0 = 1$
- (5) $x^{-n} = \frac{1}{x^n}$ Examples: $x^{-3} = \frac{1}{x^3}$; $y^{-4} = \frac{1}{y^4}$; $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
- (6) $\frac{1}{x^{-n}} = x^n$ Examples: $\frac{1}{x^{-5}} = x^5$; $\frac{1}{x^{-2}} = x^2$; $\frac{1}{3^{-2}} = 3^2 = 9$
- (7) $x^{m/n} = \sqrt[n]{x^m}$ Examples: $x^{2/3} = \sqrt[3]{x^2}$; $y^{1/2} = \sqrt{y}$

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 3.1.2. For additional examples and practice, see the Checkpoint 5A problems in the back of the textbook.

Example 1

Simplify: $(2xy^3)(5x^2y^4)$

Reorder: $2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$

Using law (1): $10x^3y^7$

Example 2

Simplify: $\frac{14x^2y^{12}}{7x^5y^7}$

Separate: $(\frac{14}{7}) \cdot (\frac{x^2}{x^5}) \cdot (\frac{y^{12}}{y^7})$

Using laws (2) and (5): $2x^{-3}y^5 = \frac{2y^5}{x^3}$

Example 3

Simplify: $(3x^2y^4)^3$

Using law (3): $3^3 \cdot (x^2)^3 \cdot (y^4)^3$

Using law (3) again: $27x^6y^{12}$

Example 4

Simplify: $(2x^3)^{-2}$

Using law (5): $\frac{1}{(2x^3)^2}$

Using law (3): $\frac{1}{2^2 \cdot (x^3)^2}$

Using law (3) again: $\frac{1}{4x^6}$

Example 5

Simplify: $\frac{10x^7y^3}{15x^{-2}y^3}$

Separate: $\left(\frac{10}{15}\right) \cdot \left(\frac{x^7}{x^{-2}}\right) \cdot \left(\frac{y^3}{y^3}\right)$

Using law (2): $\frac{2}{3}x^9y^0$

Using law (4): $\frac{2}{3}x^9 \cdot 1 = \frac{2}{3}x^9 = \frac{2x^9}{3}$

Problems

Simplify each expression. Final answers should contain no parentheses or negative exponents.

1. $y^5 \cdot y^7$

2. $b^4 \cdot b^3 \cdot b^2$

3. $8^6 \cdot 8^{-2}$

4. $(y^5)^2$

5. $(3a)^4$

6. $\frac{m^8}{m^3}$

7. $\frac{12m^8}{6m^{-3}}$

8. $(x^3y^2)^3$

9. $\frac{(y^4)^2}{(y^3)^2}$

10. $\frac{15x^2y^5}{3x^4y^5}$

11. $(4c^4)(ac^3)(3a^5c)$

12. $(7x^3y^5)^2$

13. $(4xy^2)(2y)^3$

14. $\left(\frac{4}{x^2}\right)^3$

15. $\frac{(2a^7)(3a^2)}{6a^3}$

16. $\left(\frac{5m^3n}{m^5}\right)^3$

17. $(3a^2x^3)^2(2ax^4)^3$

18. $\left(\frac{x^3y}{y^4}\right)^4$

19. $\left(\frac{6x^8y^2}{12x^3y^7}\right)^2$

20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$

21. x^{-3}

22. $2x^{-3}$

23. $(2x)^{-3}$

24. $(2x^3)^0$

25. $5^{1/2}$

26. $\left(\frac{2x}{3}\right)^{-2}$

Answers

- | | | |
|------------------------------|----------------------|-----------------------------|
| 1. y^{12} | 2. b^9 | 3. 8^4 |
| 4. y^{10} | 5. $81a^4$ | 6. m^5 |
| 7. $2m^{11}$ | 8. x^9y^6 | 9. y^2 |
| 10. $\frac{5}{x^2}$ | 11. $12a^6c^8$ | 12. $49x^6y^{10}$ |
| 13. $32xy^5$ | 14. $\frac{64}{x^6}$ | 15. a^6 |
| 16. $\frac{125n^3}{m^6}$ | 17. $72a^7x^{18}$ | 18. $\frac{x^{12}}{y^{12}}$ |
| 19. $\frac{x^{10}}{4y^{10}}$ | 20. $16x^{10}y^5$ | 21. $\frac{1}{x^3}$ |
| 22. $\frac{2}{x^3}$ | 23. $\frac{1}{8x^3}$ | 24. 1 |
| 25. $\sqrt{5}$ | 26. $\frac{9}{4x^2}$ | |

Fun Art Challenge: Use probability to create a fun design!

Be as creative as you can with this.
 Step1: Use a coin, 🍀 or ??? To get multiple outputs.
 Step2: Assign each output a design.
 Step3: Design! (Video below)

[#mathartchallenge](#)
[#mathartchallenge5](#)
[#mtbos](#) [#iteachmath](#)



💬 13 ↻ 18 ❤️ 73 ⋮



Annie Perkins @anniek_p

3d

I flipped 2 coins and assigned a crochet stitch to each permutation.

HH: single crochet

HT: half double

TH: double

TT: triple

Each row got a new stitch assigned by coin flips.

[#mathartchallenge5](#)

[#mathartchallenge](#)

